On vertex-magic total labelings of Cayley digraphs

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1. INTRODUCTION

By a digraph $G = (V, E)$ we mean a finite digraph without self loops and multiple arcs and is defined by a set V of vertices and a set E of arcs or directed edges. The set E is a subset of elements (u, v) of $V \times V$. The out-degree (or in-degree) of a vertex u of a digraph G is the number of arcs (u, v) (or (v, u)) of G and is denoted by $d+(u)$ (or $d-(u)$). A digraph G is said to be regular of out-degree d if $d+(u) = d-(u)$ for every vertex u of G. Let $|V| = p$ and $|E| = q$.

The concept of graph labeling was introduced by Rosa in 1967 [1]. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Labeled graphs serve as useful models for a broad range of applications such as coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing and data base management[12]. Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, mean labeling, arithmetic labeling etc., have been studied. Almost all of the labelings mentioned in Gallian's dynamic survey [2] deal with labelings of undirected graphs. Bloom et al. [3] defined magic labelings for directed graphs. MacDougal et al. [4] introduced the notation of vertex-magic total labeling.For a graph G with p vertices and q edges, a vertex-magic total labeling is a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, . . . , p+q\}$ such that for every vertex u □ V

(G), f(u)+f(uv) = k for some constant *k*. This constant *k* is called the magic constant of the vertex-magic total labeling. A vertex-magic total labeling is super if $f(V(G)) = \{1, 2, 3, \ldots, p\}$, we call it as *V* -super vertex-magic abeling. A graph *G* is called *V* -super vertex-magic if it admits a *V* -super vertex-magic labeling. Swaminathan and Jeyanthi [5] called a vertex-magic total labeling as a super vertex-magic total labeling if $f(E(G)) = \{1, 2, \ldots, q\}$. Thamizharasi and Rajeswari [7] studied magic labelings of cayley digraphs and its line digraphs.

An antimagic labeling of a graph with *p* vertices *q* edges is a bijection $f : E(G) \rightarrow \{1, 2, \ldots, q\}$ such that the values at the vertices are distinct, where the value of *v* is the sum of the labels on edges incident to *v*. In [13], Hartsfield and Ringel made a conjecture on vertex-antimagic labeling and Martin Baca proposed a conjecture about edge-antimagic vertex labeling [14]. Thirusangu et al. [6] studied super vertex *(a,d)* antimagic labelling and vertex magic total labelling of certain classes of cayley digraphs.

The Cayley digraph of a group provides a method of visualizing the group and its Properties. The idea of representing a group in such a manner was originated by Cayley in 1878. The Cayley graphs and Cayley digraphs are excellent models for interconnection networks [8,9,10]. Many well-known interconnection networks are Cayley digraphs.

In this paper we show the existence of *V* -super vertex-magic total labeling, *V* -super vertex-antimagic total labeling, *E*-super vertex-magic labelling, *E* -super vertex-antimagic total labelling for a certain class of Cayley digraphs.

2. PRELIMINARIES

In this section we give the basic notions relevant to this paper. Let $G = G(V, E)$ be a finite, simple, and undirected graph with v vertices and e edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers, called labels (usually the set of integers). Here, we deal with labeling with domain either the set of all vertices or the set of all edges or the set of all vertices and edges. We call these labeling the vertex labeling or the edge labeling or the total labeling, respectively.

Definition 2.1 The vertex-weight of a vertex v in G under an edge labeling to be the sum of edge labels corresponding to all edges incident with v. Under a total labeling, vertex-weight of v is defined as the sum of the label of v and the edge labels corresponding to all the edges incident with v. If all vertices in G have the same weight k, we call the labeling vertex-magic edge labeling or vertex-magic total labeling, respectively and we call k a magic constant. If all vertices in G have different weights, then the labeling is called vertex-antimagic edge labeling or vertex-antimagic total labeling, respectively.

Definition 2.2 The edge-weight of an edge e under a vertex labeling is defined as the sum of the vertex labels corresponding to every vertex incident with e. Under a total labeling, we also add the label of e. Using edge-weight; we derive edge-magic vertex or edge-magic total labeling and edge-antimagic vertex or edge-antimagic total labeling.

Definition 2.3 A digraph $G = (V, E)$ is defined by a set V of vertices and a set E of arcs or directed edges. The set E is a subset of elements (u, v) of $V \times V$. The out-degree (or in-degree) of a vertex u of a digraph G *is* the number of arcs (u, v) (or (v, u)) of G and is denoted by $d^+(u)$ (or $d^-(u)$). A digraph G *is* said to be regular of out-degree d if $d+(u) = d-(u)$ for every vertex u of G. Let $|V| = p$ and $|E| = q$.

Definition 2.4 A vertex-magic total labeling of a digraph is an assignment of integers $1, 2, \ldots, p+q$ to the vertices and the edges of G, so that at each vertex, the vertex label and the labels of its outgoing edges incident at that vertex, add to a fixed constant, called the magic constant of G. Such a labeling is V–super vertex-magic total if $f(V(G)) = \{1, 2, \ldots, p\}$ and is E-super vertex-magic total if $f(E(G)) = \{1, 2, \ldots, q\}$. A digraph that admits a V-super vertex-magic total labelling is called V-super vertex-magic. Similarly, a digraph that admits an E-super vertex magic total labeling is called E-super vertex-magic.

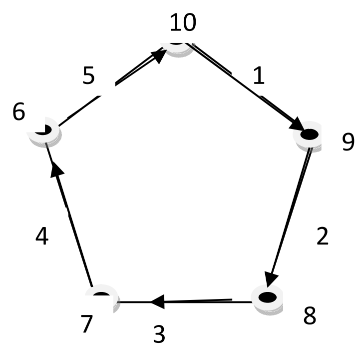

Fig. 1. *E*-super vertex magic labeling of digraph

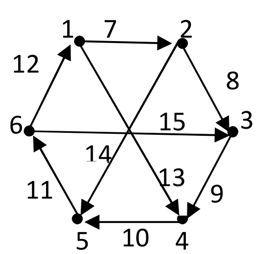

Fig. 2. *V* - super vertex anti magic labeling of a graph

Definition 2.5 A one – one map f from $V \cup E$ of a graph G onto the integers $\{1,2,3\ldots p+q\}$ is a modular vertex magic labeling if there exist a constant k such that $f(x) + \sum f(xy) = k \pmod p$, $\forall x \in V$, $0 \le k < p\text{-}1$.

Definition 2.6 A graph which admits modular super vertex magic labeling then it is called modular super vertex magic graph.

Definition. 2.7 Let G be a finite group, and let S be a generating subset of G. The Cayley digraph *Cay (G; S)* is the digraph whose vertices are the elements of G, and there is an edge from g to gs whenever $g \in G$ and $s \in S$. If $S = S^{-1}$, then there is an arc from g to gs if and only if there is an arc from gs to g.

Definition 2.8: For any natural number n, we use Z_n to denote the additive cyclic group of integers modulo n. In this section, we consider the group (Z_{2k}, A) where, A is a generating set $\{a, b, b + k\}$ with the property that

$gcd\ (a, b, k) = 1$ and either
$gcd\ (a - b, k) \neq 1$ or
$gcd\ (a, 2k) = 1$ or
$gcd\ (b, k) = 1$ or
both a & k are even or
a is odd and either b or k is odd

Definition 2.9: The line digraph of a directed graph G is the directed graph $L(G)$ whose vertex set corresponds to the arc set of G and having an arc directed from an edge e_1 to an edge e_2, if in G, the head of e_1 meets the tail of e_2.

3.MAIN RESULTS

In this section, we investigate the existence of V-Super vetex antimagic total labeling and V-Super vetex magic total labeling for cayley digraphs

3.1 V-super vertex-antimagic total labeling of cayley digraphs *Cay(G,S)*

Theorem 3.1.1 : Every Cayley digraph *Cay(G,S)* admits V - super vertex antimagic total labeling when $|S| \equiv 0 (mod2)$.

Proof: Let G be a group and the cayley digraph of the group G with generating set S be $Cay(G,S)$ with $|S| = d$ where $d \equiv 0(mod2)$. From the construction of the Cayley digraph of a group, if we have n vertices then it has dn arcs. Let $V = \{v_1, v_2, v_3,\dots,v_n\}$ and $E = \bigcup_{j=1}^{d} E_j$ i.e., $E = \{e_1^1, e_2^1, e_3^1, \dots , e_n^1\}$ U $\{ e_1^2, e_2^2, \dots, e_n^2\}$ $U \dots U$ $\{ e_1^d, e_2^d, \dots, e_n^d \}$ where e_i^j, $1 \leq j \leq d$ and $1 \leq i \leq n$ denote the out going arc from v_i generated by j^{th} generator.

Consider an arbitrary vertex $v_i \in V$. By the construction of Cayley digraph, each vertex has exactly d out going arcs each one arc is from the set E_i, $1 \leq i \leq d$.

Now, *define* $f : V\ U\ \boldsymbol{E} \rightarrow \{ 1,2,3,\dots, dn\}$ by

$$f(v_i) = i, \text{ for } 1 \leq i \leq n.$$

$$f(e_i^j) = \begin{cases} (j+1)n + 1 - f(vi), & j\ odd \\ jn + f(vi), & j\ even \end{cases}$$

Hence, $w(v_i) = f(v_i) + \sum_{j=1}^{d} f(e_i^j)$

$$= f(v_i) + n + n + 1 - f(v_i) + 2n + f(v_i) + \dots + d\,n\,f(v_i)$$

$$= (nd^2 + 2nd + d + 2\,i)/2$$

Clearly $\{ w(v_i) \,/\, 1 \leq i \leq n \}$ consists of n distinct integers. Suppose $w(v_i) = w(v_j)$ for *some* $i \neq j$, $1 \leq i,j \leq n$ then $nd^2/2 + nd + d/2 + I = nd^2/2 + nd + d/2 + j$ i.e., $i=j$, a contradiction.

Thus f is V-Super vetex anti magic total labeling of Cayley digraph, which shows $Cay(G,S)$ *is* V-Super vetex magic total when $|S| \equiv 0(mod\ 2)$.

Example 3.1.2: Consider the group $S_3 = \{ v_1 = (1)(2)(3),\ v_2 = (1)(23),\ v_3 = (12)(3),\ v_4 = (123),\ v_5 = (13)(2),\ v_6 = (132\}$, with generating set $\{v_3 = (12)(3), v_4 = (123)\}$. The Cayley digraph for the symmetric group S_3 and its V-super vertex antimagic total labeling is shown in figure 3.

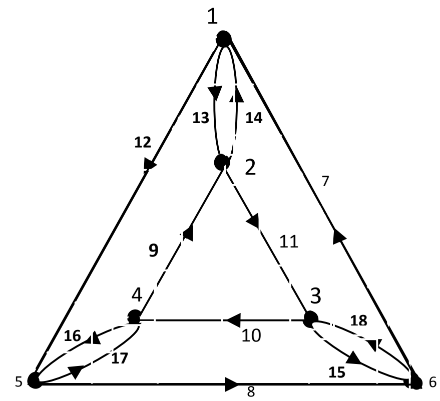

Figure 3: The V - Super Vertex antimagic total Labeling for the Cayley Digraph for the Symmetric Group S_3

3.2 *V* -super vertex-magic total labeling of cayley digraphs *Cay(G,S)*

Theorem 3.2.1 : Every Cayley digraph *Cay(G,S)* admits *V* - super vertex magic total labelling when $|S| \equiv 1 (mod 2)$.

Proof: Let *G* be a group and the cayley digraph of the group *G* with generating set *S* be *Cay(G,S)* with $|S| = d$ where $d \equiv 1 (mod 2)$. From the construction of the Cayley digraph of a group, if we have n vertices then it has *dn* arcs. Let $V = \{v_1, v_2, v_3, \dots, v_n\}$ and $E = \bigcup_{j=1}^{d} E_j$ i.e., $E = \{e_1^1, e_2^1, e_3^1, \dots, e_n^1\} \cup \{e_1^2, e_2^2, \dots, e_n^2\} \cup \dots \cup \{e_1^d, e_2^d, \dots, e_n^d\}$ where e_i^j, $1 \le j \le d$ and $1 \le i \le n$ denote the out going arc from v_i generated by j[th] generator.

Consider an arbitrary vertex $v_i \in V(G)$. By the construction of Cayley digraph, each vertex has exactly d out going arcs each one arc is from the set E_j, $1 \le j \le d$.

Now, define $f : V \cup E \rightarrow \{1,2,3,\dots, dn\}$ by

$f(v_i) = i$, for $1 \le i \le n$.

$$f(e_i^j) = \begin{cases} (j+1)n + 1 - f(vi), & j \ odd \\ jn + f(vi), & j \ even \end{cases}$$

Hence, $w(v_i) = f(v_i) + \sum_{j=1}^{d} f(e_i^j)$

$= f(v_i) + n+n + 1 - f(v_i) + 2n + f(v_i) +...+(d-1) n +f(v_i)+ (d+1) n -f(v_i)$

$= (nd^2+(2n+1)d + n + 2)/2$

Clearly $\{ w(v_i) / 1 \leq i \leq n \}$ is a constant.

Thus f is *V*-Super vetex magic total labeling of Cayley digraph, which shows *Cay(G,S) is V*-Super vetex magic total when $|S| \equiv 1(mod\ 2)$.

Example 3.2.2: Consider the group $(Z_8; 5, 3, 7)$ consisting of 8 elements where $Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and the generating set $A = \{5, 3, 7\}$.

The cayley digraph for the group $(Z_8; 5, 3, 7)$ and its *V*-super vertex magic total labeling is shown in figure 4.

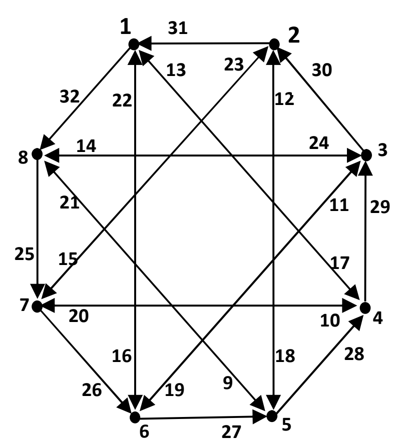

Figure 4: The V-super vertex magic total labeling for the Cayley Digraph of the Group (Z_8; 5, 3, 7)

3.3 E -super vertex-antimagic total labeling of cayley digraphs Cay(G,S)

Theorem 3.3.1 : Every Cayley digraph *Cay(G,S)* admits *E*- super vertex antimagic total labeling when $|S| \equiv 0(mod2)$.

Proof: Let *G* be a group and the cayley digraph the group *G* with generating set *S* be *Cay(G,S) with* $|S| = d$ where $d \equiv 0(mod2)$. From the construction of the Cayley digraph of a group, if we have *n* vertices then it has dn arcs. Let V =

$\{v_1, v_2, v_3,\dots,v_n\}$ *and* $E = \bigcup_{j=1}^{d} E_j$ i.e., $E = \{e_1^1, e_2^1, e_3^1, \dots , e_n^1\} \cup \{ e_1^2, e_2^2, \dots, e_n^2\} \cup \dots \cup \{ e_1^d, e_2^d, \dots, e_n^d \}$ where e_i^j, $1 \le j \le d$ *and* $1 \le i \le n$ denote the out going arc from v_i generated by j^{th} generator.

Consider an arbitrary vertex $v_i \in V$. By the construction of Cayley digraph, each vertex has exactly d out going arcs each one arc is from the set E_j, $1 \le j \le d$.

Now, define $f : V \cup E \rightarrow \{ 1,2,3,\dots, dn\}$ by

$$f(e_i^j) = \begin{cases} jn + 1 - i, & j \text{ even} \\ (j-1)n + i, & j \text{ odd} \end{cases}$$

$$f(v_i) = dn+i, \text{ for } 1 \le i \le n.$$

Hence, $w(v_i) = f(v_i) + \sum_{j=1}^{d} f(e_i^j)$

$$= dn+i +i+ n+n + 1 - i + 2n + i +\dots+(d-1) n+n+ 1-i$$

$$= (nd^2 +2nd + d +2 i)/2$$

Clearly $\{ w(v_i) /1 \le i \le n \}$ consists of n distinct integers. Suppose $w(v_i) = w(v_j)$ for *some* $i \neq j$, $1 \le i,j \le n$ then $nd^2/2 +nd + d/2 + i= nd^2/2 +nd + d/2 + j$ i.e., $i=j$, a contradiction.

Thus f is E-Super vetex anti magic total labeling of Cayley digraph, which shows $Cay(G,S)$ *is* E-Super vetex magic total when $|S| \equiv 0(mod2)$.

Example .3.2.2: Consider the dihedral group $D_8 = \{1, a, a^2, a^3, b, ab, a^2b, a^3b\}$ with the generating set $A = \{a, b\}$ such that $a^4 = b^4 =1$.

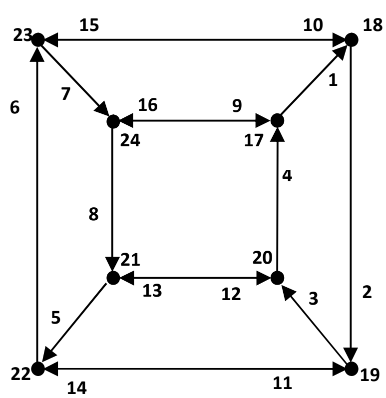

Figure 5: The *E* - Super Vertex antimagic total Labeling for the Cayley Digraph of the Dihedral Group D_8

3.4 *E* -super vertex-magic total labeling of cayley digraphs *Cay(G,S)*

Theorem 3.4.1 : Every Cayley digraph *Cay(G,S)* admits *E* - super vertex magic total labelling when $|S| \equiv 1 (mod 2)$.

Proof: Let *G* be a group and the cayley digraph the group *G* with generating set *S* be *Cay(G,S)* with $|S| = d$ where $d \equiv 1(mod2)$. From the construction of the Cayley digraph of a group, if we have n vertices then it has dn arcs. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E = \bigcup_{j=1}^{d} E_j$ i.e., $E = \{e_1^1, e_2^1, e_3^1, \ldots, e_n^1\} \cup \{e_1^2, e_2^2, \ldots, e_n^2\} \cup \ldots \cup \{e_1^d, e_2^d, \ldots, e_n^d\}$ where e_i^j, $1 \le j \le d$ and $1 \le i \le n$ denote the out going arc from v_i generated by j^{th} generator.

Consider an arbitrary vertex $v_i \in V(G)$. By the construction of Cayley digraph, each vertex has exactly d out going arcs each one arc is from the set E_j, $1 \le j \le d$.

Now, define $f : V \cup \boldsymbol{E} \rightarrow \{1,2,3,\ldots, dn\}$ by

$$f(e_i^j) = \begin{cases} (jn + 1 - i, & j \text{ even} \\ (j-1)n + i, & j \text{ odd} \end{cases}$$

$$f(v_i) = dn+i, \text{ for } 1 \le i \le n.$$

Hence, $w(v_i) = f(v_i) + \sum_{j=1}^{d} f(e_i^j)$

$= dn+i +i+ n+n + 1 - i + 2n + i +\ldots+(d-1) n+i$

$= (nd^2+(2n+1)d + n + 2)/2$

Clearly $\{ w(v_i) / 1 \le i \le n \}$ is a constant.

Thus f is E-Super vetex magic total labeling of Cayley digraph, which shows *Cay(G,S) is* E-Super vetex magic total when $|S| \equiv 1(mod\ 2)$.

The *E* - Super Vertex magic total Labeling for the Cayley Digraph of the Group $(Z_{12}; 7, 5, 11)$ is shown in the following figure 6

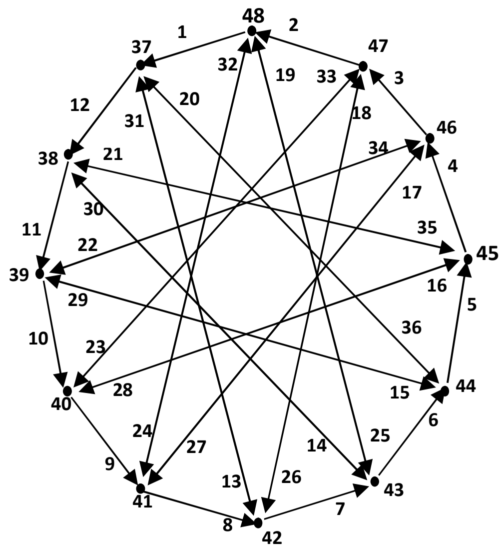

Figure 6: The The E - Super Vertex magic total Labeling for the Cayley Digraph of the Group *(Z_{12}; 7, 5, 11)*

3.7. Modular Super Vertex Magic Total Labelling labeling of cayley digraphs *Cay(G,S)*

Let $G(V,E)$ be a finite simple graph with $|V| = p$ and $|E| = q$. We extend the idea of super vertex magic total labeling to modular super vertex magic labeling.

Theorem 3.7.1: Every Cayley digraph $Cay(G,S)$ admits modular super vertex magic total labelling when $|S| \equiv 1(mod2)$.

Proof: Let G be a group and the cayley digraph the group G with generating set S be $Cay(G,S)$ with $|S| = d$ where $d \equiv 1(mod2)$. From the construction of the Cayley digraph of a group, if we have n vertices then it has dn arcs. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ *and* $E = \bigcup_{j=1}^{d} E_j$ i.e., $E = \{e_1^1, e_2^1, e_3^1, \ldots, e_n^1\} \cup \{e_1^2, e_2^2, \ldots, e_n^2\} \cup \ldots \cup \{e_1^d, e_2^d, \ldots, e_n^d\}$ where e_i^j, $1 \leq j \leq d$ *and* $1 \leq i \leq n$ denote the out going arc from v_i generated by j^{th} generator.

Consider an arbitrary vertex $v_i \in V(G)$. By the construction of Cayley digraph, each vertex has exactly d out going arcs each one arc is from the set E_j, $1 \leq j \leq d$.

Now, define $f : V \cup \boldsymbol{E} \rightarrow \{1,2,3,\ldots, dn\}$ by

For $1 \leq i \leq n.$, $1 \leq j \leq d$,

$$f(e_i^j) = \begin{cases} (jn + 1 - i, & j \text{ even} \\ (j-1)n + i, & j \text{ odd} \end{cases}$$

$$f(v_i) = dn+i, \text{ for } 1 \leq i \leq n.$$

Hence, $w(v_i) = f(v_i) + \sum_{j=1}^{d} f(e_i^j)$

$= dn+i +i+ n+n + 1 - i + 2n + i +\ldots+(d-1)\, n+i$

$= (nd^2+(2n+1)d + n + 2)/2$

Clearly $\{ w(v_i) / 1 \leq i \leq n \}$ is a constant.

Hence , $w(v_i) \equiv k(\bmod p)$ for all $v_i \in V$ for some fixed k

Thus f is Modular Super vetex magic total labeling of Cayley digraph, which shows *Cay(G,S) is* Modula Super vetex magic total when $|S| \equiv 1(\bmod 2)$.

Theorem 3.7.2: Let *G* be a graph. If *G* has a vertex magic total labeling then it has a modular super vertex magic labelling. But converse need not be true.

3.8 labeling of line digraph of cayley digraphs Cay(G,S)

Consider the Cayley digraph Cay(G,S) with *n* vertices and *dn* arcs where *d* is the number of generators. Let us denote the vertex set and the edge set of *Cay(G,S)* as as $V=\{v_1, v_2\ldots\ldots V_n\}$ and $E=\{e_{11}, e_{12}, \ldots e_{1m}, e_{21} e_{22}\ldots e_{2m}\ldots e_{n1}\ldots e_{nm}\}$ where e_{ij} is an outgoing arc from i^{th} vertex generated by the j^{th} generators. Then by the definition of the line digraph, if the Cayley digraph of a group contains *n* vertices and *nd* edges, then the corresponding line digraph *L(* ***Cay(G,S))*** contains *nd* vertices and nd^2 arcs. That is every vertex of the line digraph also has *d* incoming and *d* outgoing arcs. Hence the following theorems follows.

Theorem 3.8.1: The line digraph of the Cayley digraph *L(Cay(G,S))* admits V - super vertex-antimagic total labelling if and only if its cayley digraph admits V – super vertex antimagic labeling.

Theorem 3.8.2: The line digraph of the Cayley digraph *L(Cay(G,S))* admits V - super vertex-magic total labelling if and only if its cayley digraph admits V – super vertex magic labeling.

Theorem 3.8.3: The line digraph of the Cayley digraph *L(Cay(G,S))* admits E - super vertex-antimagic total labelling if and only if its cayley digraph admits E – super vertex antimagic labeling.

Theorem 3.8.4: The line digraph of the Cayley digraph *L(Cay(G,S))* admits E- super vertex-magic total labelling if and only if its cayley digraph admits E – super vertex magic labeling.

4 Conclusion and Scope

In this chapter, we have found V-SVIAMT , V-SVIMT , E-SVIAMT, E-SVIMT and Modular Super Vertex Magic Total labelings of different cayley digraphs and its line digraphs. This results can be extended for regular digraphs. Based on this, we suspect that every digraph may admit all the above labelings.

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